# Mathematics (Science)
## SCORING KEY

**Set: A**

**Maximum Score: 80**

| Qn. No. | | Answer Key /Value Points | Sub score | Total score |
|---|---|---|---|---|
| 1. | | For showing $R$ is reflexive | 1 | |
| | | For showing $R$ is symmetric | 1 | |
| | | For showing $R$ is transitive | 1 | |
| 2. | | $IA = A$ | 1 | |
| | | For row transformation | 1 | |
| | | $A^{-1} = \begin{bmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$ | 1 | 3 |
| 3. | i) | $AB = \begin{bmatrix} -10 & +2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$ | 2 | |
| | | $CB = \begin{bmatrix} -20 & 4 & 42 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$ | 1 | 3 |
| | | (Obtaining by doubling the first row of $AB$) | | |
| 4. | | $f(x) = (x-3)^2 + 1$ | 1 | |
| | | $y = (x-3)^2 + 1$ | | |
| | | $\sqrt{y-1} + 3 = x$ | 1 | |
| | | $f^{-1}(x) = \sqrt{x-1} + 3$ | 1 | 3 |
| 5. | | $\tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left[\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right]$ | 2 | |
| | | $= \tan^{-1}\left(\frac{3}{4}\right)$ | 1 | 3 |
| 6. | | $2\tan^{-1}(\cos x) = \tan^{-1}(2\cos x)$ | 1 | |
| | | $\frac{2\cos x}{1-\cos^2 x} = 2\cos x$ | | |
| | | $\sin^2 x = 1$ | 1 | |
| | | $x = \pm\frac{\pi}{2}$ | 1 | 3 |
| 7. | | $3x - x^2 = -2 - 8$ | 1 | |
| | | $x^2 - 3x - 10 = 0$ | 1 | |
| | | $x = 5$ or $-2$ | 1 | 3 |
| 8. | i) | $\lvert A \rvert = -28$ | 1 | |
| | ii) | $\lvert \text{adj } A \rvert = \lvert A \rvert^2$ | | |
| | | $= (-28)^2 = 784$ | 1 | |
| | iii) | $\lvert 3A \rvert = 27 \times \lvert A \rvert$ | | |
| | | $= 27 \times -28 = -756$ | 1 | 3 |

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| 9. | i) For showing commutative | 1 | |
| | For showing associative | 1 | |
| | ii) $e = \frac{1}{2}$ | 1 | |
| | iii) $a^{-1} = \frac{1}{4a}, a \neq 0$ | 1 | 4 |
| 10. | i) $(fof)\ x = f(f(x))$ | | |
| | $= f\left(\frac{2x-3}{x-2}\right)$ | 1 | |
| | $= \frac{2\left(\frac{2x-3}{x-2}\right)-3}{\left(\frac{2x-3}{x-2}\right)-2}$ | 1 | |
| | $= x$ | 1 | |
| | ii) $f(x) = \frac{x}{x+1}$ | 1 | 4 |
| 11. | $A^T = \begin{bmatrix} 1 & 2 & 1 \\ -3 & 0 & 2 \\ 1 & 4 & -2 \end{bmatrix}$ | 1 | |
| | $P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 6 \\ 2 & 6 & -4 \end{bmatrix}$ | | 1 |
| | $Q = \frac{1}{2}[A - A^T] = \frac{1}{2}\begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$ | 1 | |
| | $A = P + Q$ | 1 | 4 |
| 12. | i) $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ | 2 | |
| | ii) $A^T = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ | 1 | |
| | iii) A is symmetric since $A = A^T$ | 1 | 4 |
| 13. | $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$ | 1 | |
| | $R_1 \to R_1 + R_2 + R_3$ | | |

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| | $abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & 1 & 1\\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b}\\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1\end{vmatrix}$ | 1 | |
| | $C_2 \to C_2 - C_1$ <br> $C_3 \to C_3 - C_1$ $\quad abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & 0 & 0\\ \frac{1}{b} & 1 & 0\\ \frac{1}{c} & 0 & 1\end{vmatrix}$ | 1 | |
| | $= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ | 1 | 4 |
| 14. | i) $\frac{dx}{d\theta} = -2\sin\theta$ | ½ | |
| | $\frac{dy}{d\theta} = 3\cos\theta$ | ½ | |
| | $\frac{dy}{dx} = \frac{-3}{2}\cot\theta$ | 1 | |
| | ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ | | |
| | $= \frac{3}{2}\operatorname{cosec}^2\theta \times \frac{d\theta}{dx}$ | 1 | |
| | $= \frac{+3}{2}\operatorname{cosec}^2\theta \times \frac{1}{-2\sin\theta} = \frac{-3}{4}\operatorname{cosec}^3\theta$ | 1 | 4 |
| 15. | $y_1 = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$ | 1 | |
| | $(1+x^2)\,y_1 = 2\tan^{-1}x$ | 1 | |
| | $(1+x^2)\,y_2 + y_1\,2x = \frac{2}{1+x^2}$ | 1 | |
| | $(1+x^2)\,y_2 + 2x\,(1+x^2)\,y_1 = 2$ | 1 | 4 |
| 16. | $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) \quad 0 < x < 1$ | 1 | |
| | Put $x = \tan\theta$ | | |
| | $y = \sin^{-1}(\cos 2\theta)$ | 1 | |
| | $= \sin^{-1}\left[\sin\left(\frac{\pi}{2}\right) - 2\theta\right]$ | 1 | |
| | $= \frac{\pi}{2} - 2 \cdot \tan^{-1}x$ | | |
| | $\frac{dy}{dx} = -2 \times \frac{1}{1+x^2}$ | 1 | 4 |
| 17. | i) Area of $\Delta^{le}$ $ABC = \frac{1}{2}\begin{vmatrix}-2 & -3 & 1\\ 3 & 2 & 1\\ -1 & -8 & 1\end{vmatrix}$ | 2 | |
| | $= -15$ | | |
| | Area $= 15$ sq.units | | |

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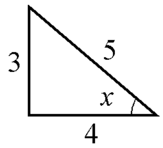
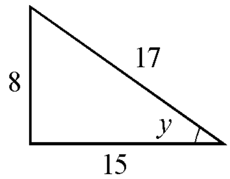
| | ii) Base line is fixex as $AB$ for the third point. Choose any value for x randomly, and find the $y$ co-ordinate for that $x$ co-ordinate accordingly. (or vice versa) | 2 | |
| | For example if we choose x = 1 | | |
| | then $= \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ 1 & y & 1 \end{vmatrix} = \pm 15$ | | |
| | Solve and find the value of y. | | |
| 18. | Let $x = \sin^{-1}\left(\frac{3}{5}\right) \Rightarrow \sin x = \frac{3}{5}$ | ½ | |
| | $y = \sin^{-1}\left(\frac{8}{17}\right) \Rightarrow \sin y = \frac{8}{17}$ | ½ | |
| | $\cos(x-y) = \cos x \cos y + \sin x \sin y$ | 1 | |
| | $\sin x = \frac{3}{5}$   | | |
| | $\cos x = \frac{4}{5}$ | ½ | |
| | $\sin y = \frac{8}{17}$   | | |
| | $\cos y = \frac{15}{17}$ | ½ | |
| | $\cos(x-y) = \frac{84}{85}$ | | |
| | $x - y = \cos^{-1}\left(\frac{84}{85}\right)$ | 1 | 4 |
| | $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ | | |
| 19. | i) $f(x) = 2x - 3$ | | |
| | one-one if $f(x_1) = f(x) \Rightarrow x_1 = x_2$ | 1 | |
| | $2x_1 - 3 = 2x_2 - 3$ | | |
| | $x_1 = x_2$ | | |
| | for showing onto | 1 | |
| | $y = 2x - 3$ | | |
| | $y + 3 = 2x, \frac{y+3}{2} = x$ | | |
| | $f^{-1}(x) = \frac{x+3}{2}$ | 1 | |
| | ii) Option (C) | 1 | |
| | iii) $f(x) = \begin{cases} \lvert x \rvert & x \le 1 \\ 2 - x & x > 1 \end{cases}$ | | |
| | [Any other corrected answer can be given full credit] | 2 | 6 |

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| 20. | $A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$ | 1 | |
| | $A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$ | 1 | |
| | $A^3 - 23A - 40I = 0$ | 1 | |
| | $A^{-1}A^3 - 23A^{-1}A - 40A^{-1}I = 0$ | 1 | |
| | ii) $A^{-1} = \dfrac{A^2 - 23I}{40}$ | 1 | |
| | $A^{-1} = \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$ | 1 | 6 |
| 21. | $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$ | 1 | |
| | $AX = B$ | ½ | |
| | $X = A^{-1}B$ | ½ | |
| | $A^{-1} = \begin{bmatrix} \frac{7}{4} & \frac{1}{4} & \frac{-3}{4} \\ \frac{-19}{4} & \frac{-1}{4} & \frac{11}{4} \\ \frac{-11}{4} & \frac{-1}{4} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$ | 3 | |
| | $X = \begin{bmatrix} \frac{7}{4} & \frac{1}{4} & \frac{-3}{4} \\ \frac{-19}{4} & \frac{-1}{4} & \frac{11}{4} \\ \frac{-11}{4} & \frac{-1}{4} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$ | | |
| | $x = 2, y = 1, z = 3$ | 1 | 6 |
| 22. | i) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right]$ | 1 | |
| | $= \sin^{-1}\left(\sin\frac{\pi}{4}\right)$ | | |
| | $= \frac{\pi}{4}$ | | |
| | ii) Option (b) | 1 | |
| | iii) Put $x = \sin\theta$ | 1 | |
| | $\sin^{-1}(2\sin\theta\ \sqrt{1-\sin^2\theta}\ )$ | | |
| | $= \sin^{-1}(\sin 2\theta)$ | 1 | |
| | $= 2\theta$ | | |
| | $= 2\sin^{-1}x$ | 1 | 6 |

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| 23. | i) $2x + \frac{xdy}{dx} + y + 2y \, . \, \frac{dy}{dx} = 0$ | 1 | |
| | $\frac{dy}{dx}(x + 2y) = -(2x + y)$ | | |
| | $\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$ | 1 | |
| | ii) $u = x^{\sin x}$ $\quad v = (\sin x)^{\cos x}$ | 1 | |
| | $y = u + v$ | | |
| | $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ | 1 | |
| | $\log u = \sin x \log x$ | | |
| | $\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} \log x \cos x$ | | |
| | $\frac{dv}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$ | 1 | |
| | $\log v = \cos x (\log \sin x)$ | | |
| | $\frac{1}{v}\frac{dv}{dx} = \frac{\cos x}{\sin x} \, . \cos x + \log(\sin x) - \sin x$ | | |
| | $\frac{dv}{dx} = (\sin x)^{\cos x} \, . \left[\frac{\cos^2 x}{\sin x} - \sin x \log(\cos x)\right]$ | 1 | 6 |
| | $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ | | |
| 24. | i) $f(x) = (x-2)^2, x \in [1, 4]$ | | |
| | $f(x)$ is continuous in $[1, 4]$ | 1 | |
| | $f(x)$ is differentiable in $(2, 4)$ | | |
| | There exists $c \in (1, 4)$ so that | | |
| | $f'(c) = \frac{f(4) - f(1)}{4 - 1}$ | 1 | |
| | $2(c-2) = \frac{4-1}{4-1} = 1$ | | |
| | $c = \frac{5}{2} \in (1, 4)$ | 1 | |
| | MVT verified | | |
| | ii) Clearly $c = \frac{5}{2}$ from first part will be the $x$-coordinate to the point of contact of tangent and the curve | | |
| | $y = (x-2)^2, \; x = \frac{5}{2}$ | | |
| | $y = \frac{5}{2} - 2$ | 1 | |
| | $y = \frac{1}{4}$ | | |
| | Point $\left(\frac{5}{2}, \frac{1}{4}\right)$ | | |

<table>
<tr><th>Qn. No.</th><th>Answer Key /Value Points</th><th>Sub score</th><th>Total score</th></tr>
<tr><td></td><td>iii) Tangent parallel to $x$ axis $\Rightarrow f'(c) = 0$<br>$2(c-2) = u$<br>$c = 2$<br>$y = (x-2)^2$<br>$y = 0$ Point (2, 0)</td><td>1<br><br><br><br>1</td><td><br><br><br><br>6</td></tr>
<tr><td>25.</td><td>i) A = {1, 2, 3, 6}<br>$a * b$ = HCF of $a$ and $b$<br>
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<tr><th>*</th><th>1</th><th>2</th><th>3</th><th>6</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
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<tr><td>3</td><td>1</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td>6</td><td>1</td><td>2</td><td>3</td><td>6</td></tr>
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ii) $e = 6$<br>
iii)<br>
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<tr><td>6</td><td>2</td><td>1</td><td>6</td><td>1</td></tr>
</table>
[write any commutative binary with 3 as identity]</td><td>3<br><br>1<br><br>2</td><td>6</td></tr>
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