FIRST YEAR HIGHER SECONDARY EXAMINATION

MARCH - 2018

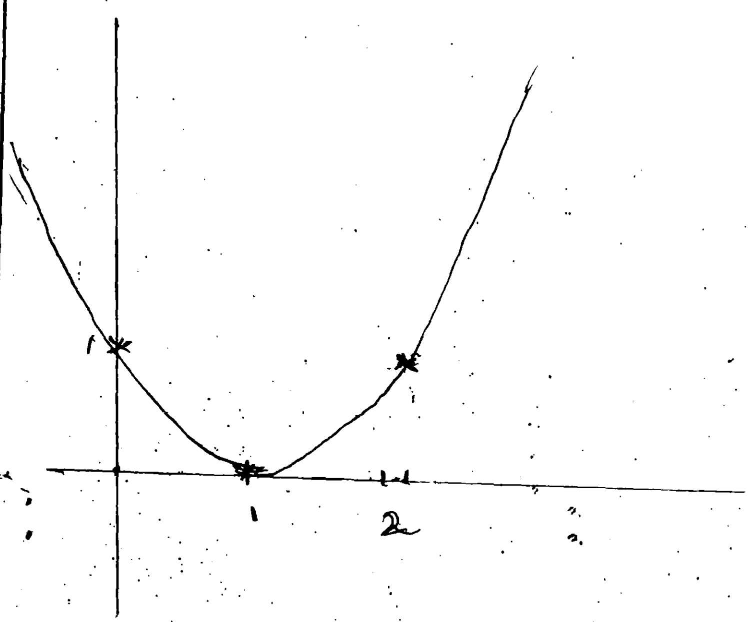
SUBJECT: MATHAMATICS (SCIENCE) Code: 118.

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
1		$S_n = 4(1+11+111+\cdots)$	1	
		$= \frac{4}{9}(9+99+999+\cdots)$	1/2	3
		$= \frac{4}{9}((10-1)+(100-1)+(1000-1)\cdots n \text{ terms})$	1/2	
		$= \frac{4}{9}(10+100+1000+\cdots n \text{ terms} - (1+1+\cdots n))$ $= \frac{4}{9}\left(\frac{10(10^n-1)}{9} - n\right)$	1	
		Remark - for the formula $S_n = \frac{a(r^n-1)}{r-1}$ - give 1 score		
2		$\sin 2x + \sin 6x - \sin 4x = 0$	1/2	
		$2\sin 4x \cos 2x - \sin 4x = 0$	1	
		$\sin 4x(2\cos 2x - 1) = 0$		
		$\sin 4x = 0$, $\cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$	1/2	3
		$4x = n\pi$, $2x = 2n\pi \pm \frac{\pi}{3}$ $x = \frac{n\pi}{4}, n \in Z$; $x = n\pi \pm \frac{\pi}{6}, n \in Z$	1	
		Remark - for the formula $\sin C \pm \sin D$ give 1 mark		
3	(1)	$P(A \text{ or } B) = P(A \cup B)$ $= P(A) + P(B) - P(A \cap B)$ $= \frac{1}{4} + \frac{1}{2} - \frac{1}{6} = \frac{7}{12}$	1	
		Remark - for formula $P(A \cup B)$ give 1 score		

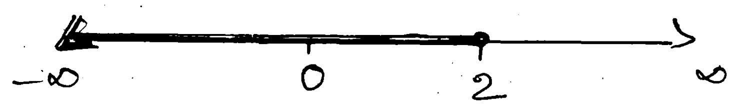
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	(ii)	P(not A and not B) $= P(A' \cap B')$	1	
		$= P(A\cup B)'$		2+1
		$= 1 - P(A\cup B)$	1/2	
		$= 1 - \frac{7}{12} = \frac{5}{12}$	1/2	
		Remark - (1) $P(A\cup B)' = 1 - P(A\cup B)$ give 1/2 score (2) For de-morgan law - 1/2 score		
4		$\frac{b-c}{b+c} = \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C} = \frac{\sin B - \sin C}{\sin B + \sin C}$	1	
		$= \frac{2\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}$	1/2	3
		$= \cot\left(\frac{B+C}{2}\right)\cdot\tan\left(\frac{B-C}{2}\right)$	1/2	
		$= \tan A/2 \cdot \tan\left(\frac{B-C}{2}\right)$	1/2	
		$= \frac{1}{\cot A/2}\cdot\tan\left(\frac{B-C}{2}\right)$	1/2	
		$\therefore \cot A/2 \cdot \frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)$		
		$\therefore \tan\left(\frac{B-C}{2}\right) = \cot A/2 \cdot \left(\frac{b-c}{b+c}\right)$		
		Remark - $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ give 1/2 score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
5	(a)	1 or (i)	1	
	(b)	$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$ $= 1 + \sin 2x$	1	3
	(c)	Maximum value of $\sin 2x = 1$ $\therefore$ Maximum value of $\sin x + \cos x = \sqrt{1+1}$ $= \sqrt{2}$ Remark :- For any answer give 1 score	1	
6	(a)	(iv) or doesn't exist Remark - give 1 score for (i) 2	1	
	(b)	$\lim_{x \to 2} \frac{x^3 - 4x^2 + 4x}{x^2 - 4} = \lim_{x \to 2} \frac{x(x^2-4x+4)}{(x+2)(x-2)}$	1	3
		$= \lim_{x \to 2} \frac{x(x-2)^2}{(x+2)(x-2)}$	1/2	
		$= \lim_{x \to 2} \frac{x(x-2)}{x+2} = 0$	1/2	
7	(i)	$P(\text{black}) = \frac{26}{52} = \frac{1}{2}$	1	
	(ii)	$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$	1	3
	(iii)	$P(\text{black face card}) = \frac{6}{52} = \frac{3}{26}$ Remark :- $P(A) = \frac{n(A)}{n(S)}$ - give 1/2 score	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
8	(i)	$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, A\}$	1	1
	(ii)	${}^{n}C_2 = 10$	1	2
		$\frac{n(n-1)}{1 \times 2} = 10$	½	
		$n^2 - n - 20 = 0$		
		$n = 5$	½	
	(iii)	$n(P(A)) = 2^5 = 32$	1	1
		Remarks: (ii) For direct answer give 2 score (iii) For any value of 'n' obtained in (ii), for correct $n(P(A))$ give full score		
9.	a)	$A = \{3, 4, 6, 10\}$	½	1
		$B = \{2, 3, 4, 5, 11\}$	½	
	(b)	$(A \cup B)' = \{1, 7, 8, 9, 12, 13\}$	1	2
		$A' = \{1, 2, 5, 7, 8, 9, 11, 12, 13\}$		
		$B' = \{1, 6, 7, 8, 9, 10, 12, 13\}$		
		$A' \cap B' = \{1, 7, 8, 9, 12, 13\}$	1	
		$\therefore (A \cup B)' = A' \cap B'$		

<table>
<tr><th>Qn. No</th><th>Sub Qns</th><th>Answer Key/Value Points</th><th>Score</th><th>Total</th></tr>
<tr><td></td><td>(c)</td><td>$n(A \cap B)' = n(U) - n(A \cap B)$
$= 13 - 2 = 11$</td><td>1</td><td>1</td></tr>
<tr><td>10</td><td>a)</td><td>(b) or (c)</td><td>1</td><td>1</td></tr>
<tr><td></td><td>(b)</td><td>(a)</td><td>1</td><td>1</td></tr>
<tr><td></td><td>(c)</td><td>
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>1</td><td>0</td><td>1</td></tr></table>Remarks: For any three correct co-ordinates give 1 score</td><td>1
1</td><td>2</td></tr>
</table>

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
11	(a)	Domain $= [-4, 4]$	1	2
		Range $= [0, 4]$	1	
	(b)	$x^2 + y^2 = 16$ $y^2 = 16 - x^2$ $y = \sqrt{16 - x^2}$ i.e. $f(x) = \sqrt{16 - x^2}$	2	2
		Remarks: for $x^2 + y^2 = 16$, give full score		
12	(a)	i (8)	1	1
	(b)	$P(1); = 1 = \frac{3^1 - 1}{2} = 1$	1	3
		$P(k) = 1 + 3 + 3^2 - - - 3^{k-1} = \frac{3^k - 1}{2}$	1	
		$P(k+1) = 1 + 3 + 3^2 - - - 3^{k-1} + 3^k$ $= \frac{3^k - 1}{2} + 3^k = \frac{3^{k+1} - 1}{2}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
13	a)	$\frac{2x-1}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$	1	
		$40x-20 \geq (15x-10-8+4x)3$		
		$40x-20 \geq 57x-54$	1	
		$-17x \geq -34$	1	4
		$x \leq 2$		
	b)		1	
14	a)	$t_n = 2n+1$	1	
	b)	$t_n = (2n+1)n^2$	1	
		$= 2n^3+n^2$		
		$S_n = \Sigma t_n$		
		$= \Sigma(2n^3+n^2)$		
		$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$	1	4
		$= \frac{n(n+1)}{2}\left[n(n+1)+\frac{2n+1}{3}\right]$		
		$= \frac{n(n+1)}{2}\left(\frac{3n^2+5n+1}{3}\right)$		
		$= \frac{n(n+1)(3n^2+5n+1)}{6}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
15		$(4-h)^2+(1-k)^2=r^2$ — ① $(6-h)^2+(5-k)^2=r^2$ — ② $16+h^2-8h+1+k^2-2k=$ $36+h^2-12h+25+k^2-10k$ $4h+8k=44$ — ③ Since centre lies on the line $4x+y=16$ $4h+k=16$ — ④ Solving ③ and ④, $k=4$, $h=3$ $\therefore r^2=(4-3)^2+(1-4)^2=10$ $\therefore r=\sqrt{10}$ Hence equ. is $(x-3)^2+(y-4)^2=10$ Remark :- For alternate method give full score. Any correct formula give one score.	 1 1 1 1	4
16)	a) b) c)	10 distance $=\sqrt{(4-4)^2+(8-0)^2+(10-0)^2}$ $=\sqrt{164}$ Since the line segment divided the yz plane, its x coordinate is zero.	1 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$\therefore \frac{mx_2+nx_1}{m+n} = 0,$	1	
		$mx_2+nx_1=0, \quad mx_2 = -nx_1$		
		$\frac{m}{n} = -\frac{x_1}{x_2} = \frac{-4}{6} = -\frac{2}{3}$		
		$\therefore m:n = -2:3.$	1	
		Remark :- b) For writing the distance formula give one score. c) For alternate method give full score.		4
17.	a)	275 is a perfect square.	1	
	b)	Let $\sqrt{2}$ is rational. $\sqrt{2} = \frac{p}{q}$, p and q have no common factor.	1	
		$p^2 = 2q^2,$		
		ie 2 divides p^2 ie 2 divides p.	1	
		$\therefore p = 2k, \quad p^2 = 4k^2$		
		$2q^2 = 4k^2, \quad q^2 = 2k^2$		
		$\therefore$ 2 divides q^2 ie 2 divides q.	1	
		$\therefore$ p and q have a common		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		factor 2. ∴ contradicts our assumption. Hence $\sqrt{2}$ is irrational.		4
18.	a)	$x^2+x+1=0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	1	
		$x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2}$	1	
	b)	Let $z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$		
		$r = \sqrt{x^2+y^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$	½	
		$\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{-1}{2}$, $\tan\theta = -\sqrt{3}$		
		$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$	½	
		$\therefore z = r(\cos\theta + i\sin\theta)$	½	
		$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$	½	
	c)	Let $\alpha = \frac{-1+i\sqrt{3}}{2}$, $\beta = \frac{-1-i\sqrt{3}}{2}$	1	6
		$\alpha^2 = \left(\frac{-1+i\sqrt{3}}{2}\right)^2 = \frac{1-i2\sqrt{3}-3}{4}$ $= \frac{-1-i\sqrt{3}}{2} = \beta$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
19.	(a)	$L_1 \Rightarrow$ Slope $= \frac{2-4}{2-0} = -1$	1	
		$\therefore$ Eqn $y-y_1 = m(x-x_1)$ $y-4 = -1(x-0)$ $x+y=4$	1	
		Remarks, writing Eqn: $y-y_1=m(x-x_1)$ give 1 score		4
		$L_2 \Rightarrow \quad y=1$	1	
		$L_3 \Rightarrow \quad y=x$	1	
		Remarks:- For any two Eqns of L_1, L_2, or L_3 give 4 score.		
	(b)	$x+y \leq 4$	1	
		$y \geq 1$ $y \leq x$	1	2
		Remarks For any two Correct Inequality give 2 score.		
20	(a)	option (i) $\left(x+\frac{1}{x}\right)^{10}$	1	
		Or		
		option (iv) $\left(x^2+\frac{1}{x}\right)^{10}$	1	
	(b)	$\left(x^2+\frac{3}{x}\right)^4 = 4C_0 x^8 + 4C_1 x^6 \cdot \frac{3}{x} + 4C_2\, x^4 \left(\frac{3}{x}\right)^2 + 4C_3\, x^2 \left(\frac{3}{x}\right)^3 + 4C_4 \left(\frac{3}{x}\right)^4$	1	
		$= x^8+12x^5+64x^2+36+\frac{81}{x}$	1	

Remarks: For writing formula for $(a+b)^n$ give 1 score.

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
20.	(c)	$T_{r+1} = (-1)^r \, 18C_r (x^2)^{18-r} \cdot \left(\frac{2}{x}\right)^r$	1	
		$= (-1)^r 2^r \cdot 18C_r \, x^{36-3r}$	1	
		for term with x^{10}		6
		$\left.\begin{array}{l} 36-3r = 10 \\ r = \frac{26}{3} \end{array}\right\}$	1	
		does not exist term contain x^{10}.		
		Remarks(i) For writing No term give -1 score.		
		writing general term of $(a+b)^n$ - give 1 score.		
21.	(a)	$a = 5, \quad b = 3 \quad , c = \sqrt{a^2-b^2} = 4$		
		$\therefore \; e = \frac{\sqrt{a^2-b^2}}{a} = \frac{4}{5}$	1	
		Focus: $(\pm ae, 0) = (\pm 4, 0)$	1	
		Remarks :- writing a and b give 1 score.		
		For any Alternate method give full score.		
	(b)	$\frac{x}{a} + \frac{y}{b} = 1$ ie $\frac{x}{5} + \frac{y}{3} = 1$	2	
		Or $3x+5y = 15$ give full score.		
		Remarks: for $\frac{x}{a}+\frac{y}{b} = 1$ give 1 score.		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
21	(c)	slope of $L = \frac{-3}{5}$	1	
		Eqn of Parallel line with slope $-3/5$ and point $(4,0)$ is		
		$y - 0 = \frac{-3}{5}(x-4)$	1	6
		ie $3x + 5y = 12$.		
		Remarks:- (1) writing Eqn of line $y - y_1 = m(x - x_1)$ give 1 score.		
		(2) For two pt formula give 1 score.		
		(3) For any alternate method and correct answer give full marks		
22.	(a)	$\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$	1	
		$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$		
		$= \lim_{h \to 0} \frac{2\cos(x + h/2) \cdot \sin h/2}{h}$	1	3
		$= \cos x$.	1	
		Remarks (1) For direct answer give 1 score.		
		(2) writing formula $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$ - 1 score.		
	(b)	$\frac{dy}{dx} = \frac{\sin x (5x^4 + \sin x) - (x^5 - \cos x)\cos x}{\sin^2 x}$		3
		$= \frac{\sin x \cdot 5x^4 - x^5 \cos x + 1}{\sin^2 x}$	1	

Remarks: (1) Eqn of Quotient rule give 1 score.
(2) For correct application of Q.R give two score.

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
23.	(a)	$12 \times {}^{(n-1)}P_3 = 5 \times {}^{(n+1)}P_3$		
		$12 \times (n-1)(n-2)(n-3) = 5 \times (n+1)\, n(n-1)$	1	
		$7n^2 - 65n + 72 = 0$	1	2
		$n = 8$ or $\frac{9}{7}$		
		Remarks: - For writing formula for ${}^nP_r$ give 1 score		
	(b)	${}^nP_r = r! \times {}^nC_r$		
		$840 = r! \times 35$		
		$r! = 24$	1	1
		$r = 4$		
		Remarks:- for formulas of ${}^nP_r$ and ${}^nC_r$ give 1 score.		
	(c)	Two diff: vowels can be selected in ${}^5C_2$ ways	1	
		Two diff: consonants can be selected in ${}^{21}C_2$ ways	1	
		Total selection 4 letters $= {}^5C_2 \times {}^{21}C_2$	1	3
		$\therefore$ Total words $= 4! \times {}^5C_2 \times {}^{21}C_2$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
24.	(a)	see table below	2	6

class	f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
10-20	6	15	90	225	1350
20-30	15	25	375	625	9375
30-40	13	35	455	1225	15925
40-50	7	45	315	2025	14175
50-60	9	55	495	3025	27225
	50		1730		68050

(a) Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1730}{50} = 34.6$ — 1

(b) Variance $\sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$

$= \frac{68050}{50} - (34.6)^2$ — 1

$= 163.84$

S.D. $= 12.8$ — 1

(c) $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{12.8}{34.6} \times 100 = 36.99$ — 1

Remarks: (1) For Eqns of $\bar{x}$, σ^2, CV, give one score.

(2) For approximate $\bar{x} = 35$, and Finding S.D and C.V give corresponding score.