

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2018

SUBJECT: MATHEMATICS COMMERCE

CODE. NO: 9053

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1	a) R_1 is reflexive	→ 1		
	b) $R_2 = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$ $(1,2), (2,3) \in R_2 \Rightarrow (1,3) \in R_2$ $\therefore R_2$ is transitive	→ (1) → (1) } 2		3
	Remark : Concept of transitive give 1 score			
2.		$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} } \rightarrow (1)$ $\vec{a} \cdot \vec{b} = 1 - 1 + 1 = 1 \rightarrow (1)$ $ \vec{b} = \sqrt{3} = \vec{a} \rightarrow (\frac{1}{2})$ $\cos \theta = \frac{1}{3} \rightarrow (\frac{1}{2})$ $\theta = \cos^{-1}(\frac{1}{3})$		3
3	a)	$l^2 + m^2 + n^2 = 1$ or $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \rightarrow (1)$ $3l^2 = 1 \rightarrow (\frac{1}{2})$ $l = \frac{1}{\sqrt{3}} \rightarrow (\frac{1}{2})$	2	3
	Remark Concept of d.c.s give $\frac{1}{2}$ score			

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6	a)	$\int (x - \frac{1}{x^2}) dx \longrightarrow (\frac{1}{2})$ $= \frac{x^2}{2} + \frac{1}{x} + C \longrightarrow (\frac{1}{2})$	1	3
	b)	$I = \int \sin x dx - \int \operatorname{cosec}^2 x dx \longrightarrow (1)$ $= -\cos x + \cot x + C \longrightarrow (1)$ Remark $\int \sin x dx = -\cos x$ or $\int \operatorname{cosec}^2 x dx = -\cot x$ give (1) score	2	
7.		$\text{Area} = \int_1^4 y dx \longrightarrow (1)$ $= \int_1^4 \sqrt{x} dx \longrightarrow (\frac{1}{2})$ $= \frac{2}{3} \left[x^{3/2} \right]_1^4 \longrightarrow (\frac{1}{2})$ $= \frac{2}{3} \left[4^{3/2} - 1^{3/2} \right] \longrightarrow (\frac{1}{2})$ $= \frac{14}{3} \longrightarrow (\frac{1}{2})$ Remark $\int_a^b f(x) dx$ or $\int_a^b y dx$ give 1 score		3

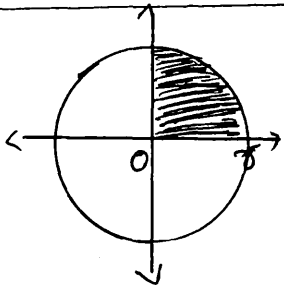
Qn No	Sub Qns	Answer Key/Value Points	Score	Total
8	a)	$f \circ f(x) = \frac{2x \left(\frac{2x-1}{4} \right) - 1}{4} \longrightarrow \frac{1}{2}$ $= \frac{2x-3}{8} \longrightarrow \frac{1}{2}$ <u>Remark</u> $f \circ f(x) = f(f(x)) \text{ give } (\frac{1}{2}) \text{ score}$	1	
	b)	$f \circ g = I \quad / \quad g \circ f = I \longrightarrow (1)$ $f(g(x)) = x \longrightarrow (\frac{1}{2})$ $f(y) = x$ $f\left(\frac{2y-1}{4}\right) = x \longrightarrow (\frac{1}{2})$ $y = \frac{4x+1}{2}$ $g(x) = \frac{4x+1}{2} \text{ is the inverse of } f \longrightarrow (1)$ $\therefore f \text{ is invertible}$ <u>Remark</u> - For any other alternate method give full score - Concept of invertible functions give 1 score	3	4

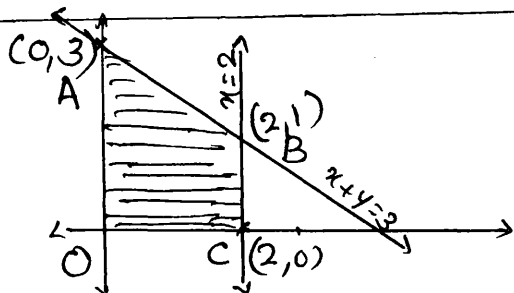
Qn No	Sub Qns	Answer Key/Value Points	Score	Total
9		$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \rightarrow (1)$ $= \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} \rightarrow (1)$ $= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} \rightarrow (1)$ $= (a-b)(b-c)(c-a) \rightarrow (1)$ <u>Remark</u> 1. Answer through direct expansion give full score 2. For direct expansion only give 1 score		4
10	a)	Let $x = \tan \theta \rightarrow (\frac{1}{2})$ $\tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) \rightarrow (\frac{1}{2})$ $= \tan^{-1} (\tan 2\theta) \rightarrow (\frac{1}{2})$ $= 2\theta \rightarrow (\frac{1}{2})$ $= \underline{\underline{2 \tan^{-1} x}}$ <u>Remark</u> 1. $\tan^{-1} \left(\frac{2x}{1-x^2} \right) = 2 \tan^{-1} x$ give 2 score 2. $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ } give 1 score or $\tan^{-1} (\tan x) = \tan x$	2	4

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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	b)	$\text{LHS} = \tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x \rightarrow (1)$ $\text{RHS} = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) = \rightarrow (1)$ $= \tan^{-1}(\tan 3\theta) \rightarrow (1)$ $= 3\theta = 3\tan^{-1}x \rightarrow (1)$ $\text{LHS} = \text{RHS}$ Remark 1. For any alternate method give full score 2. For applying $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ give (1) score	2	
11		at $x \neq 0$ $\sin x - \cos x$ is Continuous $\therefore -1$ is a constant $\therefore$ continuous } (1) at $x = 0$ $\text{LHL} = -1$ $\text{RHL} = -1$ $f(0) = -1$ $\therefore f$ is Continuous at $x = 0 \rightarrow (1)$ $\therefore f$ is Continuous on $\mathbb{R}$ Remark : Proving Cont. at $x=0$ only give full score 2. Concept of Cont. give 1 score		4

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
12	a)	$I = [\tan^{-1} x]_0^1 = \pi/4 \rightarrow (\frac{1}{2} + \frac{1}{2})$	1	4
	b)	$\int_{-4}^{-2} -(x+2) dx + \int_{-2}^4 (x+2) dx \rightarrow (1)$ $= -\left(\frac{x^2}{2} + 2x\right)_{-4}^{-2} + \left(\frac{x^2}{2} + 2x\right)_{-2}^4 \rightarrow (1)$ $= 20 \rightarrow (1)$ Remark 1) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \rightarrow (1)$ 2) For process of integration give (1) score	3	
13		$\therefore \frac{dy}{dx} = \frac{y}{x} - \operatorname{Cosec}\left(\frac{y}{x}\right) \rightarrow (\frac{1}{2})$ Put $\frac{y}{x} = v$ or $y = vx \rightarrow (1)$ $\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (1)$ $\frac{dx}{x} = -\frac{dv}{\operatorname{Cosec} v} \rightarrow (\frac{1}{2})$ $\cos v = \log x + C \rightarrow (1)$ $\cos\left(\frac{y}{x}\right) = \log x + C$		4

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
14		 Area = 4 . area of the shaded region $= 4 \int_0^r \sqrt{r^2 - x^2} dx \rightarrow (1)$ $= 4 \left[\frac{x}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \sin^{-1} \frac{x}{r} \right]_0^r \rightarrow (1)$ $= 4 \left[\frac{r^2}{2} \times \frac{\pi}{2} - 0 \right] = \underline{\underline{\pi r^2}} \rightarrow (1)$ Remark 1) Area = $\int_a^b y dx$ give 1 score 2) For writing area = πr^2 give 1 score		4
15	a)	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \rightarrow (1)$ $(x_1, y_1, z_1) = (-2, 4, -5) \rightarrow (1)$ $a, b, c = 3, 5, 6$	3	4
	b)	$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6} \rightarrow (1)$ Yes , $\frac{3+3}{3} = \frac{14-4}{5} = \frac{4+8}{6} \rightarrow (1)$ Remark For writing 'yes' only give 1 score	1	

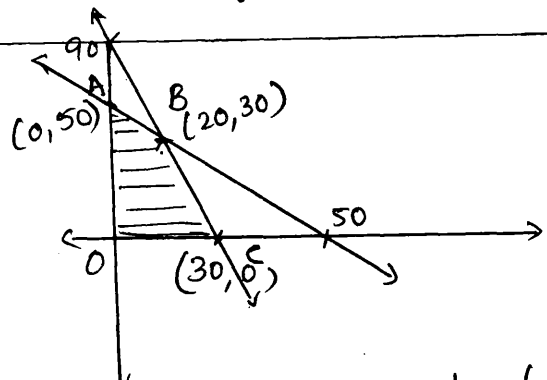
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total									
16	a)		2	4									
	b)	Corner points A(0,3), B(2,1), C(2,0), O(0,0)	1										
	c)	<table border="1" data-bbox="494 844 924 1247"><thead><tr><th>Points</th><th>$Z = 2x + y$</th></tr></thead><tbody><tr><td>A</td><td>3</td></tr><tr><td>B</td><td>5</td></tr><tr><td>C</td><td>4</td></tr><tr><td>O</td><td>0</td></tr></tbody></table> Remark 1. For drawing x and y axes give 1 Score 2. For any 3 correct corner points 1 Score	Points		$Z = 2x + y$	A	3	B	5	C	4	O	0
Points	$Z = 2x + y$												
A	3												
B	5												
C	4												
O	0												
17	a)	$m^2 + 3m^2 - m + m + m^2 + m^2 + m = 1 \rightarrow (1)$ $6m^2 + m = 1$ $m = \frac{1}{3} \rightarrow (1)$	2	4									
	b	$P(x > 4) = P(5) + P(6) = \frac{4}{9} \rightarrow 1$	1										
	c)	$P(1 < x < 4) = P(2) + P(3) = \frac{1}{3} \rightarrow 1$ Remark For writing Sum of Probability = 1 1 Score	1										

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
18	a)	$A' = \begin{bmatrix} 4 & 0 & 1 \\ 1 & 3 & -2 \\ 2 & 2 & 3 \end{bmatrix} \rightarrow (1)$	1	6
	b	$A + A' = \begin{bmatrix} 8 & 1 & 3 \\ 1 & 6 & 0 \\ 3 & 0 & 6 \end{bmatrix} \rightarrow (1)$ $A - A' = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 4 \\ -1 & -4 & 0 \end{bmatrix} \rightarrow (1)$	2	
	c)	$P = \frac{1}{2} (A + A') = \frac{1}{2} \begin{bmatrix} 8 & 1 & 3 \\ 1 & 6 & 0 \\ 3 & 0 & 6 \end{bmatrix} \rightarrow (1)$ $Q = \frac{1}{2} (A - A') = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 4 \\ -1 & -4 & 0 \end{bmatrix} \rightarrow (1)$ $A = P + Q \rightarrow (1)$ <u>Remark</u> Concept of Symmetric and Skew Symmetric matrices (1) each	3	
19	a)	Cofactor $A = \begin{bmatrix} -1 & -4 & 10 \\ 5 & -4 & -2 \\ 3 & 12 & -6 \end{bmatrix} \rightarrow (2)$ $\text{adj } A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & -4 & 12 \\ 10 & -2 & -6 \end{bmatrix} \rightarrow (\frac{1}{2})$ $ A = 24 \rightarrow (\frac{1}{2})$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$\bar{A}^{-1} = \frac{1}{24} \begin{bmatrix} -1 & 5 & 3 \\ -4 & -4 & 12 \\ 10 & -2 & -6 \end{bmatrix} \rightarrow (1)$ <u>Remark</u> 1. $\bar{A}^{-1} = \frac{\text{adj } A}{ A } \rightarrow (1)$ 2. For correct process for finding $\bar{A}^{-1}$ give 3 score	4	
	b)	$A X = B$ $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix} \rightarrow (1)$ $X = \bar{A}^{-1} B$ $= \frac{1}{24} \begin{bmatrix} -1 & 5 & 3 \\ -4 & -4 & 12 \\ 10 & -2 & -6 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix} \rightarrow (1)$ $= \frac{1}{24} \begin{bmatrix} -24 \\ 0 \\ 48 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \rightarrow (1)$ $x = -1, y = 0, z = 2$ <u>Remark</u> For finding x, y, z directly give 1 score	2	6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
20	a)	$\frac{dy}{dx} = \frac{1}{1+x^2} \longrightarrow (1)$ $\frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2} \longrightarrow (1)$ $\therefore (1+x^2)^2 \frac{d^2y}{dx^2} + 2x = 0 \longrightarrow (1)$	3	6
	b) <u>Remark</u> For alternate method give full score b) $\frac{dy}{dx} = \frac{x}{1+x^2} + \tan^{-1}x \longrightarrow (1)$ $\frac{d^2y}{dx^2} = \frac{1+x^2-2x^2}{(1+x^2)^2} + \frac{1}{1+x^2} \longrightarrow (1)$ $= \frac{2}{(1+x^2)^2} \longrightarrow (1)$	<u>Remark</u> For product rule / quotient rule give 1 score	3	
21	a)	$C(x) = 8x^2 - 7x + 4$ $C'(x) = 16x - 7 \longrightarrow (1)$ $C'(5) = 73$ $\Delta x = 0.001$ $\Delta y = C'(5) \times \Delta x = 0.073 \longrightarrow (1)$	3	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$y + \Delta y = c(5) + \Delta y = 169.073 \rightarrow (1)$ <u>Remark</u> Formula $f(x + \Delta x) = f(x) + f'(x) \cdot \Delta x$ or $\Delta y \approx \frac{dy}{dx} \Delta x$ give 1 score		6
	b	$c'(x) = 0 \Rightarrow x = \frac{7}{16} \rightarrow (1)$ $c''(x) = 16 > 0$ at $x = \frac{7}{16} \rightarrow (1)$ $\therefore c$ has a min. value at $x = \frac{7}{16} \rightarrow (1)$ <u>Remark</u> Concept of second der. test (1) score	3	
22	a)	$\vec{a} + \vec{b} = -i + j - k \rightarrow (1)$ $\vec{a} - \vec{b} = 3i - 5j + 7k \rightarrow (1)$	2	
	b	$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -\lambda & 5 \end{vmatrix} \rightarrow (1)$	1	
	c)	$[\vec{a}, \vec{b}, \vec{c}] = 0 \rightarrow (1)$ $\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -\lambda & 5 \end{vmatrix} = 0 \rightarrow (1)$ $\lambda = 3 \rightarrow (1)$	3	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total										
23	a)	 For drawing the graph ($2\frac{1}{2}$) For shading the region ($\frac{1}{2}$) <table> <tr> <th>corner points</th> <th>$Z = 3x + 2y$</th> </tr> <tr> <td>A (0, 50)</td> <td>100</td> </tr> <tr> <td>B (20, 30)</td> <td>120 $\rightarrow$ max.</td> </tr> <tr> <td>C (30, 0)</td> <td>90</td> </tr> <tr> <td>O (0, 0)</td> <td>0</td> </tr> </table> max. at $x=20, y=30$ value <u>120</u> <u>Remark</u> For each corner points give $\frac{1}{2}$ score	corner points	$Z = 3x + 2y$	A (0, 50)	100	B (20, 30)	120 $\rightarrow$ max.	C (30, 0)	90	O (0, 0)	0	3	6
corner points	$Z = 3x + 2y$													
A (0, 50)	100													
B (20, 30)	120 $\rightarrow$ max.													
C (30, 0)	90													
O (0, 0)	0													
24	a	Yes $\rightarrow$ (1) $p = \frac{1}{6}$ $q = \frac{5}{6}$ $\rightarrow$ ($\frac{1}{2} + \frac{1}{2}$)	2											
	b)	$P(X=r) = {}^7C_r \left(\frac{5}{6}\right)^{7-r} \left(\frac{1}{6}\right)^r \rightarrow \left(\frac{1}{2}\right)$		6										

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$P(x=2) = {}^7C_2 \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^2 \rightarrow (1)$ $= 21 \times \frac{5^5}{6^7} \rightarrow \text{---} \rightarrow \text{---}$ ii) $P(x=0) = {}^7C_0 \left(\frac{5}{6}\right)^7 \left(\frac{1}{6}\right)^0 \rightarrow (1\frac{1}{2})$ $= \left(\frac{5}{6}\right)^7 \rightarrow \left(\frac{1}{2}\right)$ <u>Remark</u> For writing formula $P(x=r) = {}^nC_r q^{n-r} p^r \text{ give}$ 1 score	4	

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