SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2018

SUBJECT : MATHEMATICS (SCIENCE) CODE. NO: 9018

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1.	(a)	$f\circ f(x) = f(f(x))$	½	
		$= f\left(\frac{x}{x-1}\right)$	½	
		$= \dfrac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$	½	2
		$= \dfrac{x}{x-(x-1)} = \underline{\underline{x}}$	½	
	(b)	f	1	1
		Remark: $y = \frac{x}{x-1}$ $\quad xy - y = x$ $xy - x = y$ $\quad x(y-1) = y$ $x = \frac{y}{y-1}$ $\therefore f^{-1}(y) = \frac{y}{y-1}$ or $f^{-1}(x) = \frac{x}{x-1}$	1	
2.		$A = IA$	½	
		$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$	½	
		$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ $\quad R_2 \to R_2 - 2R_1$	½	
		$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ +\frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$ $\quad R_2 \to \frac{-1}{5} R_2$	1	
		$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A.$ $\quad R_1 \to R_1 - 2R_2$	½	3
		Remark: Using matrix method $A^{-1} = \frac{\text{Adj } A}{\lvert A \rvert} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ give(1)		

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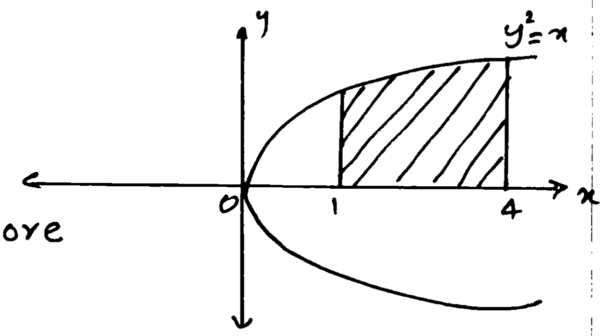
3.	(a)	(i) positive	1	1
	(b)	$f'(x) = 3x^2 - 6x + 4$	1	
		$= 3(x^2-2x+1)+1 > 0$	½	
		$\therefore f(x)$ is strictly increasing in $\mathbb{R}$	½.	2.
		Remark: If for $f'(x) = 3x^2-6x+4 > 0$, $f(x)$ is increasing (1½)		
4.	(a)	$\int_0^a f(a-x)\,dx = \int_0^a f(x)\,dx$ (iii)	1	1
	(b)	$I = \int_0^{\pi/2} \frac{\sin^4 x\,dx}{\sin^4 x + \cos^4 x}$		
		$= \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)\,dx}{\sin^4(\pi/2-x) + \cos^4(\pi/2-x)}$	1	
		$I = \int_0^{\pi/2} \frac{\cos^4(x)\,dx}{\sin^4 x + \cos^4 x}$	½	
		$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx$		
		$= \int_0^{\pi/2} dx = [x]_0^{\pi/2}$		
		$= \pi/2$	½	2
		$\therefore I = \pi/4$		
5		Area $= \int_a^b f(x)\,dx$	1	
		$= \int_1^4 \sqrt{x}\,dx$	1	
		$= \left(\frac{x^{3/2}}{3/2}\right)_1^4$	½	3
		$= \frac{14}{3}$ sq. units	½	

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
		Remark :- For this figure only give (1) score 		
6		$\frac{dy}{dx} + \frac{2}{x}y = x \log x$	½	
		$P = \frac{2}{x} \quad Q = x \log x.$	½	
		$I.F = e^{\int P dx}$	½	
		$= e^{\int \frac{2}{x} dx} = e^{2 \log x}$		
		$= \underline{x^2}$	½	
		Solutions are		
		$y.(IF) = \int Q (I.F) dx$	½	
		$y x^2 = \int x^2 . x \log x \, dx$		
		$= \int \log x . x^3 dx$		
		$= \log x . \frac{x^4}{4} - \int \frac{x^4}{x . 4} dx.$		
		$= \frac{x^4}{4} \log x - \frac{x^4}{16} + c$	½	3
7.		Let x packets of nuts and y packets of bolts		
		maximise $Z = 17.5x + 7y$	1½	
		Subject to $x + 3y \leq 12$	½	
		$3x + y \leq 12$	½	
		$x, y \geq 0$	½.	3

Remark:

	Nut	Bolts	
A	1	3	12
B	3	1	12

For this table only give (1) score

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
8	(a)	$(1,2) * (2,3) = (1+2,\ 2+3)$	½	1
		$= \underline{\underline{(3,5)}}$	½	
	(b)	$(c,d) * (a,b) = (c+a,\ d+b)$	½	1
		$= (a+c,\ b+d)$		
		$= (a,b) * (c,d)$	½	
	(c)	$(a,b) * [(c,d) * (e,f)] = (a,b) * (c+e,\ d+f)$	½	2.
		$= (a+c+e,\ b+d+f)$	½	
		$[(a,b) * (c,d)] * (e,f) = (a+c,\ b+d) * (e,f)$	½	
		$= (a+c+e,\ b+d+f)$	½	
		$\therefore (a,b) * [(c,d) * (e,f)] = [(a,b) * (c,d)] * (e,f)$		
		Remarks:		
		(i) If $a * b = b * a$ then $*$ is commutative give (½) score		
		(ii) If $a*(b*c) = (a*b)*c$ then $*$ is associative give (½) score.		
		(iii) Proving 'b' and 'c' using numbers give (3) score		
9	(a)	(ii) $\sin^{-1} x$	1	1
	(b)	Domain $[-1, 1]$	½	1
		Range $[-\pi/2,\ \pi/2]$	½	
	(c)	$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	½	2
		$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right)$	1	
		$= \tan^{-1}\left(\frac{15}{20}\right)$	½	
		$= \underline{\underline{\tan^{-1}\left(\frac{3}{4}\right)}}$		

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
10	(a)	(iii) $a^x \log a$	1	1
	(b)	$\log x^y = \log y^x$	½	
		$y \log x = x \log y$	½	
		$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y$	1	
		$\left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x}$	½	3
		$\therefore \frac{dy}{dx} = \dfrac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$	½	
11	(a)	$\frac{dy}{dx} = 2(x-2)$	½	1
		Slope $= -2$	½	
	(b)	Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$	½	
		$= \frac{4-0}{4-2} = 2$	½	
		Here $2(x-2) = 2$	½	2
		$x = 3 \quad y = 1$	½	
		Point $(3, 1)$		
	(c)	Equation of tangent is $y - y_0 = m(x - x_0)$	½	
		$y - 1 = 2(x-3)$	½	1
		$2x - y - 5 = 0$		
12.		$\int_a^b f(x)dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + \cdots + f(a+\overline{n-1}h)\right]$	1	
		$a = 0 \quad b = 2 \quad f(x) = x^2 + 1$	½	
		$h = \frac{2}{n} = \frac{b-a}{n}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$\int_0^2 x^2+1\,dx = \lim_{h\to 0} h\left[1+(h^2+1)+((2h)^2+1) + \dots + [(n-1)h]^2+1\right]$	$\frac{1}{2}$	
		$= \lim_{h\to 0} h\left[n + h^2(1^2+2^2+3^2+\dots+(n-1)^2)\right]$	$\frac{1}{2}$	
		$= \lim_{h\to 0}\left[nh + \frac{h^3 n(n-1)(2n-1)}{6}\right]$	$\frac{1}{2}$	
		$= 2 + \frac{2(2-0)(4-0)}{6}$		
		$= 2 + \frac{8}{3} = \frac{14}{3}$	$\frac{1}{2}$	4
		Remark:		
	i	For Direct method $\int_0^2 (x^2+1)dx = \left(\frac{x^3}{3}+x\right)_0^2 = \frac{14}{3}$ give (1) score		
	ii	$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+\dots + f(a+(n-1)h)\right]$ and $\int_0^2 x^2+1\,dx = \frac{14}{3}$ give (4) score		
13	(a)	point of intersection $x^2+x^2=50$ $\quad x^2=25$ $x=\pm 5$ $\quad y=\pm 5$ $\therefore$ Point $P(5,5)$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
13	b	Required Area $= \int_0^5 x\,dx + \int_5^{\sqrt{50}} \sqrt{50-x^2}\,dx$	1	
		$= \left(\frac{x^2}{2}\right)_0^5 + \left[\frac{x}{2}\sqrt{50-x^2} + \frac{50}{2}\sin^{-1}\frac{x}{\sqrt{50}}\right]_5^{\sqrt{50}}$	1	
		$= \frac{25}{2} + \frac{25\pi}{2} - \frac{25}{2} - 25.\frac{\pi}{4}$	½	
		$= \frac{25\pi}{4}$	½	3
		<u>Remark.</u>		
		(i) Area $= \frac{\pi r^2}{8} = \frac{\pi \times 50}{8} = \frac{25\pi}{4}$ (3)		
		(ii) Area $= \frac{1}{2}\int_0^{\sqrt{50}} \sqrt{50-x^2}\,dx.$		
		$= \frac{25\pi}{4}$ (3)		
		(iii) Area $= \int_a^b f(x)\,dx$ (1)		
14.	(a)	(iii) 2	1	1
	(b)	$\frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$	1	
		$\int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$		
		$\log \tan x = -\log \tan y + c$	2	3
		$\log \tan x + \log \tan y = c$		

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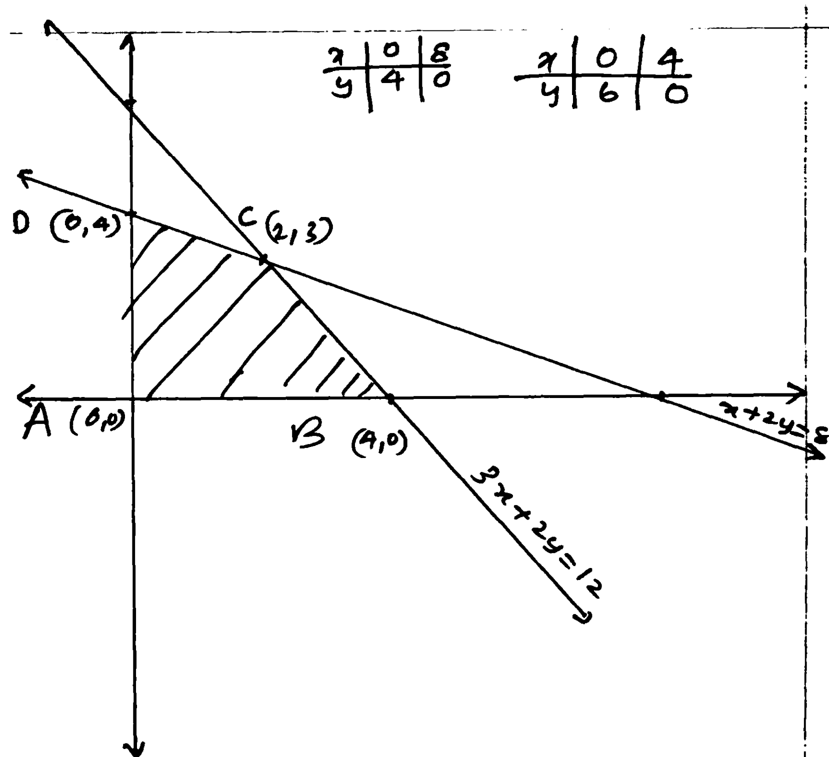
15	a.	$[\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}+\bar{a}] = (\bar{a}+\bar{b}) \cdot [(\bar{b}+\bar{c}) \times (\bar{c}+\bar{a})]$	½	
		$= (\bar{a}+\bar{b}) \cdot [\bar{b}\times\bar{c}+\bar{b}\times\bar{a}+\bar{c}\times\bar{c}+\bar{c}\times\bar{a}]$	½	
		$= (\bar{a}+\bar{b}) \cdot [\bar{b}\times\bar{c}+\bar{b}\times\bar{a}+\bar{c}\times\bar{a}]$	½	
		$= \bar{a}\cdot\bar{b}\times\bar{c} + \bar{a}\cdot\bar{b}\times\bar{a} + \bar{a}\cdot\bar{c}\times\bar{a}$ $+ \bar{b}\cdot\bar{b}\times\bar{c} + \bar{b}\cdot\bar{b}\times\bar{a} + \bar{b}\cdot\bar{c}\times\bar{a}$	1	
		$= 2[\bar{a}\ \bar{b}\ \bar{c}]$	½	3
	b.	$[\bar{a}\ \bar{b}\ \bar{c}] = 0$ Hence $\bar{a}, \bar{b}, \bar{c}$ are coplanar	1	1
16.	a.	$\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ $6x+3y+2z=6$	2.	2
	b.	Plane parallel to given plane is $6x+3y+2z=k$.	1	
		Since it passes through 1, 2, 3 $k=18$	½	
		∴ equation of plane is $6x+3y+2z=18$	½	2
		Remark. Equation of plane is $6(x-1)+3(y-2)+2(z-3)=0$ $6x+3y+2z-18=0$ (2) score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
17.		$\begin{array}{c\|c\|c} x & 0 & 8 \\ \hline y & 4 & 0 \end{array}$ $\begin{array}{c\|c\|c} x & 0 & 4 \\ \hline y & 6 & 0 \end{array}$ 	2.	
		At ~~At~~ $z = -3x+4y$ $A(0,0)$ $z = 0$ $B(4,0)$ $z = -12$ minimum $C(2,3)$ $z = 6$ $D(0,4)$ $z = 16$ z is minimum at $B(4,0)$	2	4
18.	(a)	$2x + ^{-}y = 10$ $3x + y = 5$ $x = 3$ $y = -4$	1½ ½	2
	(b)	$A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$P = \frac{1}{2}\left[A + A^T\right]$		
		$= \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$, symmetric	1	
		$Q = \frac{1}{2}\left[A - A^T\right]$		
		$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ skew, symmetric	1	
		$A = P + Q$	1	4
19.	a	$\begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix} = 0$	2.	2
		Remark.		
		$\Delta = \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$ $R_2 \to R_2 - R_1$		
		$= 0$ (2) score		
		For direct expantion give (1) score		
	b (i)	Proving $AB = I$	$1\frac{1}{2}$	
		$\therefore B = A^{-1}$	$\frac{1}{2}$	
		Remark : $A^{-1} = \frac{Adj\,A}{\lvert A \rvert} = B$ give (2) score.		

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	ii	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	½	
		$X = A^{-1}B$	½	
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	½	
		$= \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$	½	4
		$x = 0 \quad y = 5 \quad z = 3$		
20	(a)	$f(x) = \cos x \quad g(x) = x^2$, both are continuous	1	
		Composition of two continuous functions are continuous	½	
		$\therefore f(g(x)) = fog(x) = \cos(x^2)$ is continuous	½	2
	(b)(i)	$\frac{dy}{dx} = a e^{a\cos^{-1}x} \times \frac{-1}{\sqrt{1-x^2}}$		
		$= \frac{-a e^{a\cos^{-1}x}}{\sqrt{1-x^2}}$	1	1
	ii	$\sqrt{1-x^2}\,\frac{dy}{dx} = -ay.$	½	
		$(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2.$	½	

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		$(1-x^2)\, 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \times -2x = 2a^2 y\frac{dy}{dx}$	1	
		$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$	1	3
21	a	$-\frac{\cos mx}{m} + c$	1	1
	b	$x^2+2x+2 = (x+1)^2+1$	1	
		$\int\frac{dx}{\sqrt{x^2+2x+2}} = \int\frac{dx}{\sqrt{(x+1)^2+1}}$	1	
		$= \log\lvert(x+1)+\sqrt{x^2+2x+2}\rvert + c$	1	3
	c	$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$	½	
		$A = -1 \qquad B = 2$	½	
		$\int\frac{x}{(x+1)(x+2)}dx = \int\frac{-1}{x+1}dx + \int\frac{2}{x+2}dx$	½	
		$= -\log(x+1) + 2\log(x+2) + c$	½	2
22	(a) i	$\bar{a}+\bar{b} = 4\bar{i}+4\bar{j}$	1	2
		$\bar{a}-\bar{b} = 2\bar{i}+4\bar{k}$	1	
	(ii)	unit vector $\perp^r$ to both $= \dfrac{\begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}}{\sqrt{16^2+16^2+8^2}}$	1	

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		$= \dfrac{16\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{576}}$	$\frac{1}{2}$	
		$= \dfrac{2\hat{i} - 2\hat{j} - k}{3}$	$\frac{1}{2}$	2
		Remark: Unit vector $\perp$ to $\bar{a}$ and $\bar{b}$ $= \dfrac{\bar{a} \times \bar{b}}{\lvert \bar{a} \times \bar{b} \rvert}$ give (1) score		
	(b)(i)	$\bar{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$	$\frac{1}{2}$	1
		$\bar{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$	$\frac{1}{2}$	
	(ii)	$\therefore \bar{AB} = \bar{BC}$	$\frac{1}{2}$	1
		A, B, C are collinear	$\frac{1}{2}$	
		Remark: Using distance formula or Section formula for proving Collinearity give (1) score.		
23	(a)	Cos $a_1 = 2$ $b_1 = 5$ $c_1 = -3$; $a_2 = -1$ $b_2 = 8$ $c_2 = +4$	$\frac{1}{2}$	
		$\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$	$\frac{1}{2}$	
		$= \dfrac{-2+40-12}{\sqrt{4+25+9}\ \sqrt{1+64+16}}$	$\frac{1}{2}$	
		$= \dfrac{26}{9\sqrt{38}}$	$\frac{1}{2}$	2.

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		Remark: $\cos\theta = \frac{\bar{a}\cdot\bar{b}}{\lvert a\rvert \lvert b\rvert}$ give (1/2) score.		
	(b)	$a_1 = \hat{i}+2\hat{j}+3\hat{k}$ $b_1 = \hat{i}-3\hat{j}+2\hat{k}$ $a_2 = 4i+5j+6k$ $b_2 = 2\hat{i}+3\hat{j}+\hat{k}$	1	
		Shortest Distance $= \left\lvert \frac{(a_2-a_1)\cdot(b_1\times b_2)}{\lvert b_1\times b_2\rvert} \right\rvert$	1	
		$a_2-a_1 = 3\hat{i}+3\hat{j}+3\hat{k}$	1/2	
		$b_1\times b_2 = -9\hat{i}+3\hat{j}+9\hat{k}$	1	
		$SD = \frac{-27+9+27}{\sqrt{171}}$	1/2	
		$= \frac{9}{\sqrt{171}} = \frac{3}{\sqrt{19}}$		4
24	(a)	$\sum P(x) = 1$	1	1
	b (i)	$k+2k+3k+4k+5k+7k+8k+9k+10k+11k+12k = 1$	1	
		$72k = 1$	1/2	2
		$k = 1/72$	1/2	
	(ii)	$P(x>3) = P(x=4)+P(x=5)$	1/2	
		$= \frac{11}{72}+\frac{12}{72} = \frac{23}{72}$	1/2	1

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	(iii)	$P(-3<x<4)$		
		$= P(x=2) + P(x=-1) + P(x=0) + P(x=1) + P(x=2) + P(x=3)$	½	1
		$= 43k = \underline{\underline{\frac{43}{72}}}$	½	
	(iv)	$P(x<-3)$	½	
		$= P(x=-5) + P(x=-4)$		1
		$= 3k = \underline{\underline{\frac{3}{72}}}$	½	
		<u>Remark:</u> For any value of 'k' if (ii), (iii) and (iv) are correct give corresponding scores.		