SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2019

SUBJECT : MATHEMATICS (COMMERCE) CODE. NO: SY 51

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1.	i)	$f(x_1) = f(x_2) \implies 3-4x_1 = 3-4x_2$	1/2	
		$\implies -4x_1 = -4x_2$	1/2	
		$\implies x_1 = x_2$		
		$\therefore$ f is one-one.		
		Let $y \in R$, $y = f(x)$	1/2	
		$\implies y = 3-4x$		
		$\implies x = \frac{3-y}{4} \in R$	1/2	
		$\therefore$ f is onto.		
	ii)	$y = 3-4x$	1/2	3
		$\implies x = \frac{3-y}{4}$.	1/2	
		$\therefore f^{-1}(x) = \frac{3-x}{4}$. or $f^{-1} = \frac{3-y}{4}$.		
		Remark. for defenition of 1-1 and onto give 1/2 score each. For any alternate correct method give full score.		
2.		$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$	2	
		$= \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$	1	3
		Remark: For any 9 correct entries give full score.		

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3.		Area $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	1	
		$= \frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 2 & 0 & 1 \\ 4 & 5 & 1 \end{vmatrix}$	1	3
		$= \frac{1}{2}\left\| 0-3(-2)+1(10) \right\|$	½	
		$= 8$ sq. units	½	
4.	(i)	$f(2) = 3$.	1	
	(ii)	Since f is continuous at $x=2$		
		$\lim_{x\to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$	1	3
		$4k = 3$	½	
		$k = 3/4$	½	
		Remark: For definition of continuity give 1 score.		
5	(i)	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$.	1	
	(ii)	$\int \frac{dx}{x^2-6x-7} = \int \frac{dx}{(x-3)^2-4^2}$	½	3.
		$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\| + C$	1	
		$= \frac{1}{8}\log\left\|\frac{x-7}{x+1}\right\| + C$	½	

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6	(i)	$\vec{a}\cdot\vec{b} = 0$	1	
	(ii)	Let $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$		
		$\vec{a}\cdot\vec{b} = 3 + 1 + 3 = 7$	½	
		$\lvert\vec{a}\rvert = \sqrt{1+1+9} = \sqrt{11}$	½	
		$\lvert\vec{b}\rvert = \sqrt{9+1+1} = \sqrt{11}$	½	3
		$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \dfrac{7}{\sqrt{11}\cdot\sqrt{11}}$	½	
		$= \dfrac{7}{11}$		
		<u>Remark</u>: For $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ give ½ score		
7.		$\vec{a_1} = \hat{i} + \hat{j} + \hat{k}$ $\quad \vec{b_1} = \hat{i} + \hat{j}$		
		$\vec{a_2} = 2\hat{i} + \hat{j} + 2\hat{k}$ $\quad \vec{b_2} = \hat{j} + \hat{k}$	1	
		$\vec{a_2} - \vec{a_1} = \hat{i} + \hat{k}$	½	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$	½	
		$(\vec{a_2} - \vec{a_1})\cdot(\vec{b_1} \times \vec{b_2}) = 1 + 1 = 2$	½	3
		$\lvert\vec{b_1} \times \vec{b_2}\rvert = \sqrt{3}$		
		$S.D = \left\lvert \dfrac{(\vec{a_2} - \vec{a_1})\cdot(\vec{b_1} \times \vec{b_2})}{\lvert\vec{b_1} \times \vec{b_2}\rvert} \right\rvert = \dfrac{2}{\sqrt{3}}$	½	
		<u>Remark</u>: For formula give 1 score		

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8	(i)	$2*3 = 2\times 3^2 = 18$	1	
	ii)	$a*b = ab^2$, $b*a = ba^2$	1/2	
		$\Rightarrow a*b \neq b*a$	1/2	
		$\Rightarrow *$ is not commutative		
	iii)	$a*(b*c) = a*(bc^2)$	1/2	
		$= a(bc^2)^2 = ab^2c^4$	1/2	4
		$(a*b)*c = (ab^2)*c$	1/2	
		$= ab^2c^2$	1/2	
		$a*(b*c) \neq (a*b)*c$		
		$\Rightarrow *$ is not associative		
		Remark (i) For direct answer or example give full score ii) For direct answer give 1 score, with example give full score.		
9	i	(a) $\pi/6$	1	
	ii)	Let $\sin^{-1}\left(\frac{5}{13}\right) = x \Rightarrow \sin x = \frac{5}{13}$	1	
		$\Rightarrow \tan x = \frac{5}{12}$		
		$\therefore x = \tan^{-1}\frac{5}{12}$.		
		$||^{ly}$ $\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$	1	

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		$\therefore \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4} = \tan^{-1}\left(\dfrac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right)$ $= \tan^{-1}\frac{56}{33}$	1	4
		Remark: For any correct alternate method give full score. For any suitable formula give 1 score.		
10	i)	0	1	
	ii)	$R_1 \rightarrow R_1+R_2+R_3$ $\Rightarrow \begin{vmatrix} 3x+k & 3x+k & 3x+k \\ x & x+k & x \\ x & x & x+k \end{vmatrix}$	1	
		$\Rightarrow (3x+k)\begin{vmatrix} 1 & 1 & 1 \\ x & x+k & x \\ x & x & x+k \end{vmatrix}$	1/2	
		$C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$ $\Rightarrow (3x+k)\begin{vmatrix} 1 & 0 & 0 \\ x & k & 0 \\ x & 0 & k \end{vmatrix}$	1	4
		$\Rightarrow (3x+k)\cdot 1\cdot(k^2-0) = k^2(3x+k)$	1/2	
		Remark For correct expansion give one score.		

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11	i)	$f'(x) = 2x - 4$	1	
	ii)	$f(x)$ is continuous in $[1, 3]$	1/2	
		$f'(x) = 2x - 4$ exist in $(1, 3)$	1/2	
		$f(1) = 0$ and $f(3) = 0$	1/2	
		$\Rightarrow f(1) = f(3)$		4
		Conditions satisfied	1	
		$\therefore f'(c) = 0$		
		$\Rightarrow 2c - 4 = 0 \Rightarrow c = 2 \in (1, 3)$	1/2	
		Hence Rolle's theorem Verified		
12	(i)	(b) 0	1	
	ii)	Let $I = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x}\, dx \rightarrow (1)$		
		we have $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$	1	
		$\therefore I = \int_0^{\pi/2} \frac{\cos^n(\pi/2 - x)}{\sin^n(\frac{\pi}{2} - x) + \cos^n(\frac{\pi}{2} - x)}\, dx$	1/2	
		$= \int_0^{\pi/2} \frac{\sin^n x}{\cos^n x + \sin^n x}\, dx \rightarrow (2)$	1/2	4.
		(1)+(2) $\Rightarrow 2I = \int_0^{\pi/2} 1\,dx = (x)_0^{\pi/2} = \frac{\pi}{2}$	1/2	

$\therefore \underline{I = \pi/4}$ 1/2

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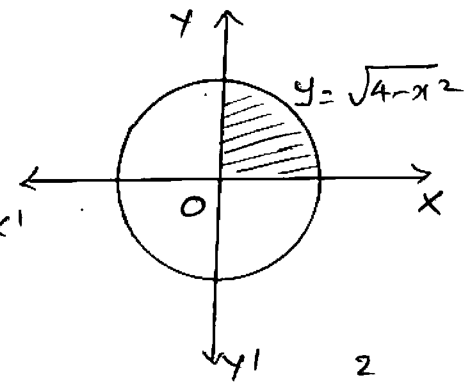
13.		Area under a curve $= \int_a^b y\, dx$	½	



		Area of the shaded portion $= \int_0^2 \sqrt{4-x^2}\, dx$	1	
		$\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$	1	4.
		$\therefore$ Area $= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\cdot\sin^{-1}\frac{x}{2}\right]_0^2$	½	
		$= \left(0 + 2\sin^{-1}(1)\right) - (0+0)$		
		$= 2\cdot\frac{\pi}{2} = \pi$ sq. units	½	
		$\therefore$ Required Area $= 4\pi$ sq. units	½.	
		Remark: For correct figure give ½ Score.		
14	(i)	Order − 1 degree − 1	½ + ½	
	(ii)	$P = 1/x$, $Q = x^2$	½	
		Integrating factor $= e^{\int P dx} = e^{\int 1/x\, dx} = e^{\log x}$.	½ ½	

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
		$\therefore IF = x$.		
		Solution is		
		$y \cdot IF = \int Q\cdot(I.F)\,dx$	1	4.
		i.e. $yx = \int x^2 \cdot x\,dx = \int x^3 dx$.		
		$= \frac{x^4}{4} + c$	½	
15	(i)	$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$	1	
		$= 5\hat{i} + \hat{j} - 4\hat{k}$	1	
	ii)	$\hat{n} = \frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}$, $\lvert\vec{a}\times\vec{b}\rvert = \sqrt{42}$	½ + ½	
		$= \frac{5\hat{i}+\hat{j}-4\hat{k}}{\sqrt{42}}$.		4
	(iii)	Area of Parallelogram $= \lvert\vec{a}\times\vec{b}\rvert$	1	
		$= \sqrt{42}$ sq. units		
		Remark:- For $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ give ½ Score.		

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16	(i)	$P(A\cap B) = P(A)+P(B)-P(A\cup B)$	1	
		$= 0.6+0.5-0.8 = 0.3$	½	
		$P(A/B) = \frac{P(A\cap B)}{P(B)}$	1	
		$= \frac{0.3}{0.5} = 3/5 = 0.6$	½	4
	(ii)	(d) $P(E\cap F)$	1	
17		Let food M be x kg and food N be y kg.	1	
		Minimise $Z = 50x+70y$	1	
		Subject to $3x+4y \geq 9$	1	4.
		$5x+2y \geq 11$	1	
		$x, y \geq 0$		
18	(i)	$A^T = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$	1	
	(ii)	Let $P = \frac{A+A^T}{2}$	½	
		$= \frac{1}{2}\begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	½	
		$P^T = P \Rightarrow P$ is symmetric.		

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		$Q = \frac{A - A^T}{2}$	½	
		$= \frac{1}{2}\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$	½	
		$Q^T = -Q \Rightarrow Q$ is skew-symmetric.	1	
		Also $\underline{\underline{P + Q = A}}$		
	(iii)	$A \cdot A^T = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$	1	6.
		$= \begin{bmatrix} 5 & 7 & 2 \\ 7 & 14 & 1 \\ 2 & 1 & 2 \end{bmatrix}$	1	
19	(i)	$\lvert A \rvert = 3(2-3) + 2(4+4) + 3(-10)$	1	
		$= -3 + 16 - 30 = \underline{\underline{-17}}$		
	(ii)	Cofactor matrix $= \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$	2	
		$\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	1	

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
	(iii)	$AX = B$, $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$		
		$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$	1/2	
		$X = A^{-1}B$	1/2	
		$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$	1	6.
		$\therefore$ ~~x y~~ $x = 1,\ y = 2,\ z = 1$		
20	(i)	$x^y = y^x$		
		Take log on both sides	1	
		$\log x^y = \log y^x$		
		$y \log x = x \log y$.		
		differentiating		
		$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y \cdot 1$	1	
		$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}$.	1	
		$\therefore \frac{dy}{dx} = \frac{\log y - y/x}{\log x - x/y}$		

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	(ii)	$\frac{dx}{dt} = 4at, \quad \frac{dy}{dt} = 4at^3$	1+1	
		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at}$	½+½	6
		$= \underline{\underline{t^2}}$		
21	(i)	$y^2 = x$	½	
		$2y\frac{dy}{dx} = 1$	½	
		$\frac{dy}{dx} = 1/2y$	½	
		Slope, $m = \frac{dy}{dx}$ at $(1,1)$		
		$= 1/2$.		
		Equation of tgt. is	1	
		$y - y_1 = m(x - x_1)$	½	
		i.e. $y - 1 = 1/2(x-1)$		
		$\Rightarrow \underline{\underline{x - 2y + 1 = 0}}$		
	(ii)	$f'(x) = 6x^2 - 6x - 36$	1	
		$f'(x) = 0 \Rightarrow 6(x^2 - x - 6) = 0$		
		$\Rightarrow (x-3)(x+2) = 0$	1	6
		$\Rightarrow x = -2, 3$		
		for increasing $f'(x) \geq 0$, decreasing $f'(x) \leq 0$		
		increasing in the interval $(-\infty, -2] \cup [3, \infty)$	½	

and decreasing in $(-2, 3)$ ½

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22.	(i)	Since vectors are coplanar	1	
		$[\vec{a}\ \vec{b}\ \vec{c}] = 0$		
		$\therefore \begin{vmatrix} 2 & -1 & 1 \\ 3 & 1 & 2 \\ 1 & \lambda & -3 \end{vmatrix} = 0$	1	
		$\Rightarrow 2(-3-2\lambda)+1(-9-2)+1(3\lambda-1) = 0$	1/2	
		$\Rightarrow \lambda = -18$	1/2	
	(ii)	$[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}]$		
		$= (\vec{a}+\vec{b})\cdot[(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})]$	1	
		$= (\vec{a}+\vec{b})\cdot\{\vec{b}\times\vec{c}) + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a}\}$	1/2	
		$= (\vec{a}+\vec{b})\cdot[\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a}]$	1/2	
		$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a}) + \vec{b}\cdot(\vec{c}\times\vec{a})$	1/2	6
		$= [\vec{a}\ \vec{b}\ \vec{c}] + 0 + 0 + 0 + 0 + [\vec{b}\ \vec{c}\ \vec{a}]$	1/2	
		$= 2[\vec{a}\ \vec{b}\ \vec{c}]$		

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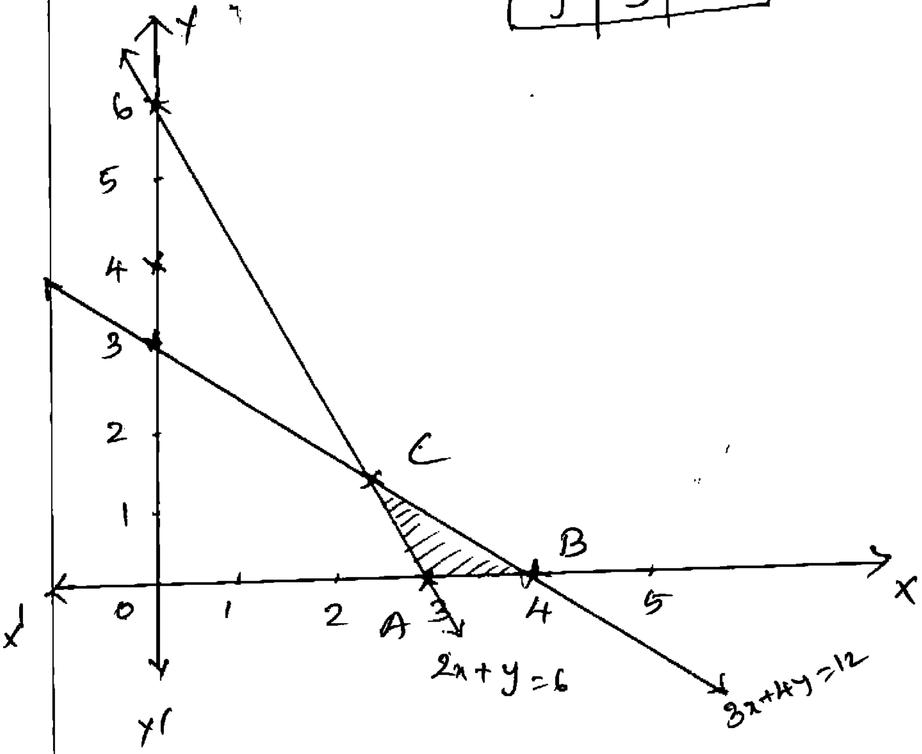
23	(i)	$2x+y=6$. $\begin{array}{c\|c\|c} x & 0 & 3 \\ \hline y & 6 & 0 \end{array}$	½	
		$3x+4y=12$ $\begin{array}{c\|c\|c} x & 0 & 4 \\ \hline y & 3 & 0 \end{array}$	½	
			3.	
	(ii)	Corner points are $A(3,0)$ $B(4,0)$ $C\left(\frac{12}{5}, \frac{6}{5}\right)$	1½.	6
		A $(3,0)$ $Z=30$ B $(4,0)$ $\boxed{Z=40}$ $\Longrightarrow$ Maximum. C $\left(\frac{12}{5}, \frac{6}{5}\right)$ $Z=28.8=\frac{144}{5}$		
		Maximum at $x=4$, $y=0$.	½	
<u>Remark</u>:		For each correct line give 1 score. Incorrect shaded region and correct graph } give 3½ score.		

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24	(i)	$\sum P_i = 1$	1	
		$\therefore k + 2k + 2k + 2k + k = 1$		
		$8k = 1$	1	
		$k = 1/8$		
	(ii)	$X : 0 \quad 1 \quad 2 \quad 3 \quad 4$		
		$P(x) : 1/8 \quad 2/8 \quad 2/8 \quad 2/8 \quad 1/8$		
		$x_i P_i(x) : 0 \quad 2/8 \quad 4/8 \quad 6/8 \quad 4/8$	1/2	
		$x_i^2 P_i(x) : 0 \quad 2/8 \quad 8/8 \quad 18/8 \quad 16/8$	1/2	
		Mean $= \sum x_i P_i = \frac{16}{8} = 2$	1	
		$\sum x_i^2 P_i = \frac{44}{8} = 5.5$		
		Variance $= \sum x_i^2 P_i - \left(\sum x_i P_i\right)^2$	1	6.
		$= 5.5 - 4$	1	
		$= 1.5$		